FOURIER ANALYSIS OF THE NUMBER OF PUBLIC LAWS 1789 - 1976

David L. Farnsworth, Eisenhower College
Michael G. Stratton, GTE Sylvania

1. Introduction. In an earlier study (reference [1], hereafter referred to as EFF) the number of public laws (NPL) for each United States Congress was analyzed using moving averages, linear regression and similar methods. A centered five Congress moving average of NPL which had been subsequently detrended (called cyclic numbers in EFF) was used as the dependent variable. Three distinct eras of different levels of productivity of laws were perceived and examined separately. Various political variables such as the President's percent of the popular vote and the percent of the Senate which is Republican were treated as independent variables and their degree of correlation with the processed NPL measured.

Here Fourier analysis methodology (see [2]) is used to re-examine the behavior of NPL over time.

There has been some debate concerning whether the number of public laws has meaning [1]. The actual count of NPL is unquestioned since the identity of public laws is clear. Roughly speaking a public law is what one usually thinks of as a law passed by Congress and not vetoed by the President. Other types of actions which require voting by Congress are private laws and internal business such as votes on adjournment. A private law might, for example, allow one particular person to immigrate.

In a sense we are examining the quantity of output of laws alone and trying to make some order out of it. In this study we ignore any independent variables - political, economic, social or others. We are assuming that each law has the same weight in our count or that the total NPL for each Congress is metric data.

The goal herein is to look for periodic components of NPL over time through Fourier decomposition. This is an independent means of validating whether the eras are meaningful or not. This is done without massaging the data or using any preconceived or a priori notions.

We assume that there is stability in the system. That is, regardless of changes such as those in committee structures in Congress, the advent of social legislation, etc., the overall political structure is unchanged over all 94 Congresses. Hence, the output (i.e. NPL) can be inquired into as a single data set.

2. Decomposition into Eras. The present authors agree with EFF in its decomposition of the Congresses into three eras based upon an examination of the five Congress moving average of NPL. The three eras can be extended to include all Congresses with no transition periods between eras. The three eras in EFF are: Era 1 - Congresses 3-34, Era 2 - Congresses 41-66, and Era 3 - Congresses 70-85. For example, the NPL for Congresses 35 and 36 are each within one (Era 1) standard deviation of the mean NPL of Era 1, but the NPL for Congresses 37 through 40 are each more than five (Era 1) standard deviations from that mean. On the other hand, the NPL for Congresses 35 and 36 are both more than 2.8 (Era 2) standard deviations from the mean NPL of Era 2, and all of Congresses 37 through 40 have NPL within about one (Era 2) standard deviation of the mean NPL of Era 2.

The extended Era 1 is composed of Congresses 1 through 36. This adds two Congresses onto each end of the EFF Era 1. These two were deleted in EFF as an artifact of the five Congress moving average. Similarly, the new Era 2 commenses with Congress 37 and ends with 66. Era 3 begins with Congress 67 and ends with the last Congress (94). There is no apparent reason for deletion of the more modern Congresses 86 through 94 from the third era. In particular the NPL of each of these nine Congresses are within 1.3 (Era 3) standard deviations of the mean NPL of Era 3. However, the downtrend since the 84th Congress should be noted.

3. Autocorrelation in Each Era. The authors are indebted to Ms. P. Paolotto of Colgate University for her comments and initial calculations which form the basis of this section. The eras of EFF are used here.

The von Neumann or Durbin - Watson ratio of lag one measures autocorrelation and also randomness [3]. This statistic in its non-circular form for data $x_1, x_2, \ldots, x_n$ is

$$\frac{\sum_{i=1}^{n-1} (x_{i+1} - x_i)^2}{\sum_{i=1}^{n-1} (x_i - \bar{x})^2}$$

and ranges from zero to four. The value of two represents randomness or no autocorrelation.

The circular autocorrelation coefficient r with lag one enjoys the exact relationship $v = 2(1-r)$ with the circular von Neumann ratio v with lag one. This relationship holds approximately for the non-circular definitions. We utilize the von Neumann ratio instead of the equivalent autocorrelation coefficient.

The von Neumann ratios for the NPL of Eras 1, 2 and 3 are 1.55, 1.42 and 1.75 respectively. For the residuals of the linear least-squares fit of NPL the ratios are 1.93, 1.61 and 1.73. None are significantly different from 2.00 at level 0.10. However, for the cyclic numbers of

EFF the ratios are 0.33, 0.51 and 1.32. The von Neumann ratio is significantly different from 2.00 at any reasonable level for Eras 1 and 2 and falls in the inconclusive region at level 0.10 for Era 3 [3]. Not surprisingly a great deal of autocorrelation was introduced by the moving average procedure.

4. Building the Models. For each data set the periodogram was produced using the fast Fourier transform. This required first subtracting the mean from each data point and extending the data with the appropriate number of zeros. The Fourier frequency with largest percent of the sum of ordinates was adjusted using the Brent iterative procedure. The data was then changed to be the residuals obtained by deleting this new frequency. The periodogram of these residuals was produced and the Fourier frequency with the largest percent of the sum of ordinates was chosen. The previously found (adjusted) frequency and this second frequency were fit at the same time by the Brent procedure to the original data. The two new frequencies were deleted from the data. This procedure was continued until one of our stopping conditions was reached.

Our stopping conditions were (1) five frequencies having been fit, (2) a von Neumann ratio for the residuals being sufficiently different from two for the eras [4], and (3) the periodogram of the residuals being just that of noise.

The data was linearly detrended if a period longer than the data set emerged in the original periodogram.

A number of components beyond those presented herein were investigated. By overfitting the data more confidence was gained regarding the choice of model[4]. One symptom of overfitting is the failure of the Brent procedure to converge with even as many as one hundred iterations. Usually only a few iterations were necessary.

Most of our computer programs are adaptations of those found in Bloomfield [2]. We chose not to taper the data since leakage did not affect our procedure.

This is a fitting exercise rather than smoothing. This type of fit has the advantage of linearity in the sum of the sinusoidal terms. One frequency can be considered at a time so that its impact can be weighed separately. The procedure and the programs used show high resolution, that is, frequencies close together can be distinguished and remain stable as more frequencies are introduced.

5. Model for All 94 Congresses. The mean of all 94 Congress' NPL is 430.6 laws and the sum of squares is 7.93×10^6 (standard error 294). The non-circular von Neumann ratio of lag one is 0.22 which indicates autocorrelation. However, this can be traced to the use of an overall mean which forces the denominator to be large. Each era has little autocorrelation as shown in Sections 3 and 7. The linear least-squares fit has a von Neumann ratio of 0.94 and its correlation coefficient is 0.877.

The model developed by our procedure is

$$\begin{aligned}NPL = {} & 9.39\,N - 20.25 + 60.38\cos(.084(N-1)) \\ & - 35.46\sin(.084(N-1)) \\ & -45.09\cos(.197(N-1)) + 55.82\sin(.197(N-1)) \\ & -41.37\cos(.396(N-1)) + 35.30\sin(.396(N-1)) \\ & +10.29\cos(.673(N-1)) + 53.18\sin(.673(N-1)) \\ & +27.21\cos(1.510(N-1)) - 33.95\sin(1.510(N-1)).\end{aligned}$$

N is the Congress number. Its residual sum of squares is 0.93×10^6 (standard error 100). The von Neumann ratio is 1.61.

It is interesting to examine each frequency separately. An $\omega = 0.084$ radians corresponds to a period of 74.8 Congresses. This is simply a modification of the linear trend. An $\omega = 0.197$ corresponds to a 31.9 Congress period and is a rough indication of the eras. These two frequencies together comprise forty eight percent of the sum of amplitudes of the five components.

The remaining three components have periods of 15.9, 9.3 and 4.2 Congresses. In a five Congress moving average smoothing their amplitudes would be multiplied by 0.85, 0.60 and 0.17 respectively.

Evaluating this model at N=95 and 96 for a forecast of NPL for the present and the next Congress yields the estimates 750 and 840 laws with standard error of 100. That is, the current down swing in NPL will be reversed.

6. Models for Each EFF Era. Models for each EFF era and some of their statistics are displayed in Table 1. The periods range from 2.5 to 24.2 Congresses. The models are graphed in Figure 2.

Since the analysis in EFF of the relationship between certain political variables as independent variables and NPL as dependent variable was performed using linear regression techniques (see Section 1), column 1 of Table 2 suggests that the strengths of the relationships may be different if NPL rather than cyclic numbers were used. However, the r's for Eras 1 and 2 are significantly different from zero at the 0.02 level. The unstarred r's in Table 2 are not significant at the 0.05 level. Of course, using the wider eras described above would also affect these.

The second column of Table 2 displays r's which measure the degree of fitting of the models. The residual sum of squares of Table 1 does also.

The third column of Table 2 is interesting. Since the residuals of the models in this report are produced by fitting and the cyclic numbers (residuals) of EFF are yielded partly by a smoothing operation, the correlations would not necessarily be high. However, in Era 3 there is a period of 4.9 Congresses (ω=1.286) which is destroyed by the EFF five Congress moving average. Hence, this was also a fit in a sence.

No forecast is possible for the 94th Congress since Era 3 stops at the 85th Congress.

Figure 1 is a graph of this model. The choices of cut off points at Congresses 36-37 and 66-67 of the extended eras in Section 2 are reinforced by this model. The same eras are perceived by Fourier analysis as were chosen by the simpler criterium of the number of standard deviations from local means (Section 2).

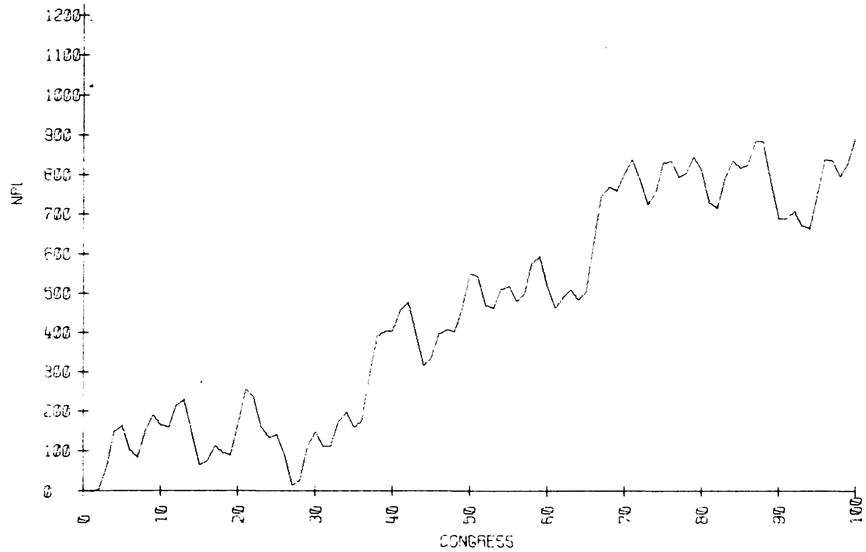

FIGURE 1: MODEL OF NPL FOR ALL 94 CONGRESSES (PLOTTED THROUGH THE 100TH CONGRESS)

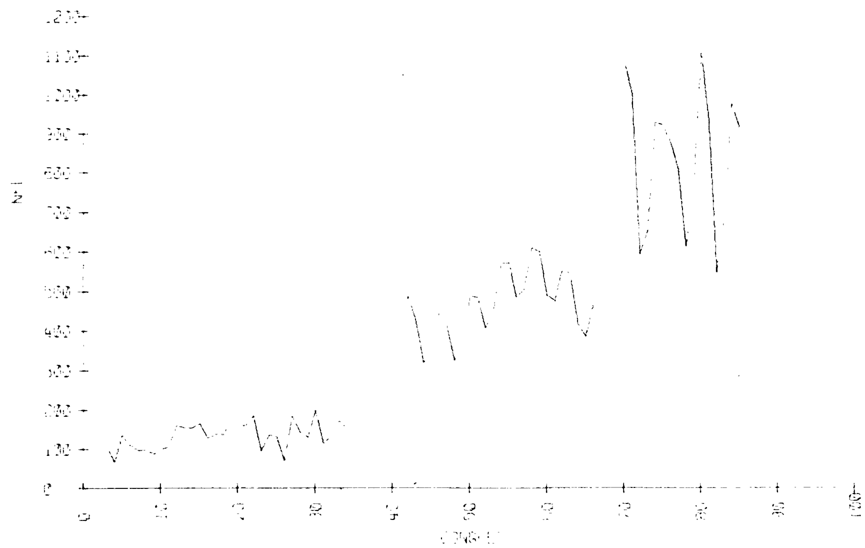

FIGURE 2: MODELS OF NPL FOR EACH EFF ERA

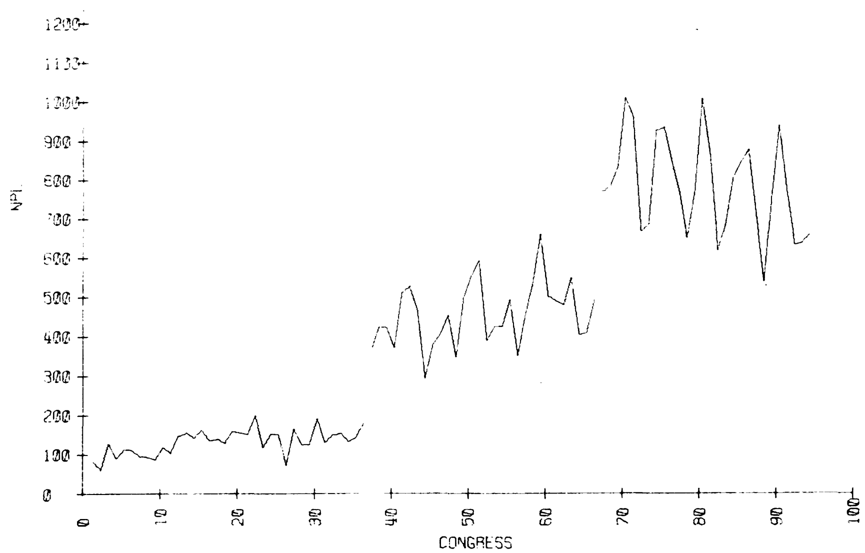

FIGURE 3: MODELS OF NPL FOR EACH FULL ERA

Era	Congresses	NPL Mean	NPL Sum of Squares	NPL Std. Error	NPL von Neumann	Model Residual Sum of Squares	Model Std. Error	Model von Neumann	Cyclic No. Residual Sum of Squares	Cyclic No. Std. Error	Cyclic No. von Neumann
1	3-34	134.3	4.31×10^4	38	1.55	0.63×10^4	14	1.56	0.76×10^4	16	0.33
2	41-66	464.7	2.98×10^5	111	1.42	1.35×10^5	75	2.18	0.55×10^5	48	0.51
3	70-85	839.8	5.77×10^5	203	1.75	1.02×10^5	85	1.60	0.20×10^5	37	1.32

Models Era 1:

$$\begin{aligned} NPL = {} & 1.84\,N + 104.17 - 14.54\,\mathrm{cox}\,(.294(N-3)) - 15.04\sin(.294(N-3)) - 9.14\cos(.816(N-3)) \\ & + 19.92\sin(.816(N-3)) + 17.63\cos(2.081(N-3)) - 0.51\sin(2.081(N-3)) \\ & + 6.29\cos(2.270(N-3)) - 17.90\sin(2.270(N-3)) - 4.42\cos(2.527(N-3)) \\ & - 18.07\sin(2.527(N-3)) \end{aligned}$$

Era 2:

$$\begin{aligned} NPL = {} & 466.62 - 26.55\cos(.260(N-41)) - 84.48\sin(.260(N-41)) - 22.17\cos(1.547(N-41)) \\ & + 69.77\sin(1.547(N-41)) \end{aligned}$$

Era 3:

$$\begin{aligned} NPL = {} & 821.43 + 179.48\cos(1.286(N-70)) + 81.80\sin(1.286(N-70)) + 84.53\cos(1.898(N-70)) \\ & + 87.57\sin(1.898(N-70)) \end{aligned}$$

N is the Congress number.

TABLE 1: MODELS AND STATISTICS FOR THE EFF ERAS

Era	Congresses	NPL Mean	NPL Sum of Squares	NPL Std. Error	NPL von Neumann Ratio	Model Residual Sum of Squares	Model Std. Error	Model von Neumann Ratio
1	1-36	132.1	4.90×10^4	38	1.45	1.22×10^4	19	1.64
2	37-66	456.8	3.14×10^5	106	1.41	0.99×10^5	60	2.09
3	67-94	786.3	8.00×10^5	175	1.39	3.70×10^5	119	1.57

Models Era 1:

$$\begin{aligned} NPL = {} & 1.79\,N + 103.50 - 8.49\cos(.262(N-1)) - 16.34\sin(.262(N-1)) - 16.48\cos(.745(N-1)) \\ & + 5.31\sin(.745(N-1)) + 12.72\cos(2.128(N-1)) - 10.14\sin(2.128(N-1)) - 3.26\cos(2.314(N-1)) \\ & - 15.52\sin(2.314(N-1)) - 8.62\cos(2.572(N-1)) - 16.20\sin(2.572(N-1)) \end{aligned}$$

Era 2:

$$\begin{aligned} NPL = {} & 4.33\,N + 236.81 - 67.20\cos(.686(N-37)) + 26.99\sin(.686(N-37)) + 23.39\cos(1.506(N-37)) \\ & + 70.99\sin(1.506(N-37)) + 11.45\cos(3.217(N-37)) + 33.07\sin(3.217(N-37)) \end{aligned}$$

Era 3:

$$\begin{aligned} NPL = {} & -5.58\,N + 1243.12 - 123.90\cos(1.242(N-67)) - 90.88\sin(1.242(N-67)) + 26.31\cos(1.990(N-67)) \\ & + 63.46\sin(1.990(N-67)) \end{aligned}$$

N is the Congress number.

TABLE 3: MODELS AND STATISTICS FOR THE EXTENDED ERAS

	Paired Models		
	NPL & EFF Cyclic No.	NPL & Residuals of the Models	EFF Cyclic No. & Residuals of Models
Era 1 (n=32):	0.424*	0.442*	0.181
Era 2 (n=26):	0.501*	0.672*	0.369
Era 3 (n=16):	0.135	0.420	0.661*

TABLE 2: CORRELATION COEFFICIENTS AMONG THREE REPRESENTATIONS OF NPL (*means $\alpha \leq 0.02$)

7. Models for Each Extended Era. Models for each extended era and some of their statistics are displayed in Table 3. Figure 3 is a graph of all three eras. A comparison of Figures 2 and 3 shows that nearly every peak and valley in the two graphs correspond. Hence, the shorter EFF eras are indeed subsets of the full eras.

In Era 1 there are two longer periods of 24.0 and 8.4 Congresses. Era 2 possesses two large periods of 9.2 and 4.2 Congresses. The longer period in Era 3 is 5.0 Congresses. The remaining era periods are fewer than 3.2 Congresses. Each era can be characterized not only by its level of productivity but also by its set of frequencies or periods. Each of these longer periods arise approximately in the model of all 94 Congresses in Section 5.

Evaluating the model for Era 3 at N = 95 & 96 for a forecast of NPL for the next two Congresses yields the estimates 824 and 878 laws with standard error 119. Hence, this model also predicts an upswing in NPL.

8. Summary. A model was constructed by Fourier decomposition of the time series of NPL for all 94 Congresses. Eras of the level of productivity of public laws were discerned. The three eras are Congresses 1-36, Congresses 37-66, and Congresses 67-94. The three eras of a previous study (EFF) are subsets of these eras.

Models were constructed for each EFF era and each of the full eras.

The model for all 94 Congresses and the model for the most recent full era predict a reversal in the present downtrend in NPL.

References

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